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NUMERICAL STUDY OF DOUBLE-DIFFUSIVE, FREE CONVECTIVE FLOW PAST A MOVING VERTICAL CYLINDER

P. Loganathan and P. Ganesan UDC 536.25

The double-diffusive, free convective flow past a moving semi-infinite vertical cylinder has been studied numerically. The mass, momentum, energy, and species concentration equations have been solved by a finite-difference method using an implicit scheme of Crank–Nicolson type. The finite-difference scheme is unconditionally stable and accurate. Graphical results for the velocity, temperature, concentration, local and average skin friction, and Nusselt and Sherwood numbers are illustrated and discussed for various physical parametric values. The role of temperature stratification in the ambient medium has been analyzed.

Introduction. Many free convective processes occur in the environment with temperature stratification. Good examples are closed containers and environmental chambers with heated walls. Of interest is also free convection associated with heat-rejection modules where the ocean environment is stratified. Free convective flows driven by temperature or concentration differences have been studied extensively. When both temperature and concentration differences occur simultaneously, the free convective flow can become quite complex. Gebhart and Pera have provided an excellent overview of this field and have indicated the importance of these flows in engineering systems and in nature [1]. They obtained a similarity solution for free convective flow from a vertical surface on account of buoyancy created by temperature and concentration differences. For laminar free convection along a vertical plate, Cheesewright has obtained similarity solutions with various types of nonuniform ambient temperature distributions [5] by using the techniques of Yang et al. [2], who showed that the similarity solution was not possible for natural convection from an isothermal vertical plate to a stable thermally stratified ambient. Chen and Eichhorn studied natural convection from a vertical surface to a thermally stratified fluid for both stratified and unstratified cases with the stratification parameter ranging from 0.6 to 4.5 [4]. A finite-difference solution of the double-diffusive, free convection problem can handle more complex cases. Srinivasan and Angirasa solved the problem of laminar natural convection flow resulting from the combined buoyancy effects of thermal and mass diffusion over a semi-infinite vertical surface immersed in a thermally stratified medium by employing the finite-difference scheme of the explicit method [3].

In recent years, the convective heat transfer about cylindrical heated bodies has begun to attract more attention owing to the fact that cylinders have been used in nuclear waste disposal, energy extraction in underground, and catalytic beds. However, to the best of the author's knowledge, the thermal stratification effect has never been considered for a moving vertical cylinder, and this fact motivates the present investigation. In this work, the problem of free convective flow over a moving semi-infinite vertical cylinder immersed in a thermally stratified medium is analyzed. This kind of problem may find applications in engineering and in geophysical flows.

Mathematical Formulation. A problem of two-dimensional, laminar free convective flow of a viscous incompressible fluid past a semi-infinite vertical cylinder with constant temperature and mass diffusion in a thermally stratified medium is analyzed. The x-axis is taken along the axis of the cylinder and the radial coordinate r is normal to the cylinder at the leading edge. Initially, it is assumed that the cylinder and the fluid are at the same temperature $T'_{\infty, x}$ and the concentration of the fluid is equal to C'_{∞}. It is also assumed that when the time t' becomes larger than zero the temperature and the concentration near the cylinder are raised to T'_{w} and C'_{w}, respectively, and later they are maintained at the same level. Under the above assumptions, the governing boundary-layer equations for mass, momentum, energy, and species concentration for free convective flow with the Boussinesq approximation are as follows:

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$$\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = 0 , \tag{1}$$

$$\frac{\partial u}{\partial t'} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = g\beta (T' - T'_{\infty,x}) + g\beta^* (C' - C'_\infty) + \frac{\upsilon}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right), \tag{2}$$

$$\frac{\partial T'}{\partial t'} + u\frac{\partial T'}{\partial x} + v\frac{\partial T'}{\partial r} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T'}{\partial r}\right), \tag{3}$$

$$\frac{\partial C'}{\partial t'} + u\frac{\partial C'}{\partial x} + v\frac{\partial C'}{\partial r} = \frac{D}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C'}{\partial r}\right). \tag{4}$$

The appropriate initial and boundary conditions for the velocity, temperature, and concentration are

$$t' \le 0: \quad u = 0 , \quad v = 0 , \quad T' = T'_{\infty,x} , \quad C' = C'_\infty \text{ for all } x \ge 0 \text{ and } r \ge r_0 ;$$

$$t' > 0: \quad u = u_0 , \quad v = 0 , \quad T' = T'_{\text{w}} , \quad C' = C'_{\text{w}} \text{ at } r = r_0 ,$$

$$u = 0 , \quad T' = T'_\infty , \quad C' = C'_\infty \text{ at } x = 0 \text{ and } r \ge r_0 , \tag{5}$$

$$u \to 0 , \quad T' \to T'_{\infty,x} , \quad C' \to C'_\infty \text{ as } r \to \infty .$$

Equations (1)–(4) and conditions (5) are similar to those in [7], where a free convective boundary layer with chemically reactive species diffusion was considered. Introduction of the dimensionless quantities

$$X = \frac{x\upsilon}{u_0 r_0^2} , \quad R = \frac{r}{r_0} , \quad U = \frac{u}{u_0} , \quad V = \frac{v r_0}{\upsilon} , \quad t = \frac{t'\upsilon}{r_0^2} , \quad C = \frac{C' - C'_\infty}{C'_{\text{w}} - C'_\infty} , \quad T = \frac{T' - T'_{\infty,x}}{T'_{\text{w}} - T'_{\infty,0}} ,$$

$$\text{Gc} = \frac{g\beta^* r_0^3 (C'_{\text{w}} - C'_\infty)}{\upsilon u_0} , \quad \text{Gr} = \frac{g\beta (T'_{\text{w}} - T'_{\infty,0}) r_0^3}{\upsilon u_0} , \quad \text{Sc} = \frac{\upsilon}{D} , \quad \text{Pr} = \frac{\upsilon}{\alpha} , \quad S = \frac{1}{T'_{\text{w}} - T'_{\infty,0}} \frac{dT'_{\infty,x}}{dX} \tag{6}$$

reduces Eqs. (1)–(4) to the form

$$\frac{\partial (RU)}{\partial X} + \frac{\partial (RV)}{\partial R} = 0 , \tag{7}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial R} = \text{Gr}\, T + \text{Gc}\, C + \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial U}{\partial R}\right), \tag{8}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial R} + SU = \frac{1}{\text{Pr}\, R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right), \tag{9}$$

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial R} = \frac{1}{\text{Sc}\, R}\frac{\partial}{\partial R}\left(R\frac{\partial C}{\partial R}\right). \tag{10}$$

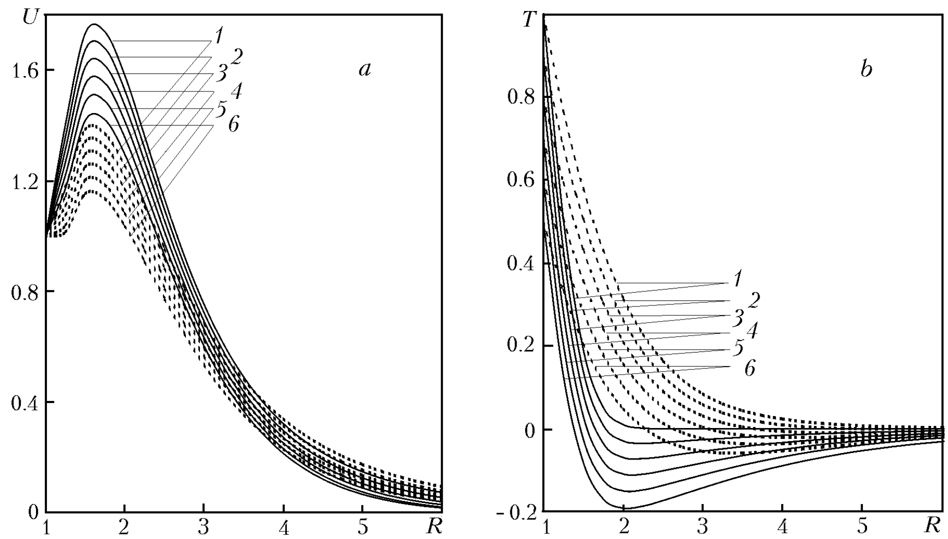

Fig. 1. Transient velocity (a) and temperature (b) profiles at X = 1 for Gr = Gc = 5, Sc = 0.6, and Pr = 0.71 (solid curves) and 7 (dashed curves) with different values of S: 1) S = 0; 2) 0.1; 3) 0.2; 4) 0.3; 5) 0.4; 6) 0.5.

The corresponding initial and boundary conditions for dimensionless quantities are given as

$$t \le 0: \quad U = 0\,, \quad V = 0\,, \quad T = 0\,, \quad C = 0 \text{ for all } X \ge 0 \text{ and } R \ge 1\,;$$

$$t > 0: \quad U = 1\,, \quad V = 0\,, \quad T = 1 - SX\,, \quad C = 1 \text{ at } R = 1\,,$$

$$U = 0\,, \quad T = 0\,, \quad C = 0 \text{ at } X = 0\,,$$

$$U \to 0\,, \quad T \to 0\,, \quad C \to 0 \text{ at } R \to \infty\,. \tag{11}$$

The numerical procedure in solution of the nonlinear coupled equations (7)–(10) under conditions (11) is analogous to that used in [7]. We proved the stability and compatibility of the finite-difference scheme, which ensures convergence of the numerical solution.

Results and Discussion. The effect of the presence of thermal stratification in the ambient is shown in Figs. 1 and 2 for the aiding flow. The velocity profiles in Fig. 1a show that an increase in stratification decreases the velocity. This is obvious since ambient thermal stratification decelerates the buoyancy-driven flows. A decrease in U causes diffusion to dominate over convection. Hence, the concentration boundary layer thickens as the stratification parameter increases (Fig. 2).

It should be noted that flow reversal occurs at a high ambient temperature, i.e., a rising hot fluid cools, appears in the environment with a high temperature, and then sinks downwards. It is seen from Fig. 1b that the presence of high thermal stratification can cause a negative dimensionless temperature. This is because the fluid near the cylinder can have a temperature lower than the ambient. We note that temperature for Pr = 7.0 is higher than in the case of Pr = 0.71. The reason is that the fluid with Pr = 7.0 has a lower thermal diffusivity than that with Pr = 0.71. Hence, the fluid with Pr = 7.0 will tend to exchange less heat with the surrounding fluid by diffusion. Thus, the cold fluid coming from below will tend to retain its low temperature.

Knowing the numerical values of velocity, temperature, and concentration, we now calculate the local and average skin friction (shear stress) and the rates of heat and mass transfer both in the transient and steady states. The local as well as average skin friction and Nusselt and Sherwood numbers in terms of dimensionless quantities are given by

$$\tau_X = -\,(\partial U/\partial R)_{R=1}\,, \tag{12}$$

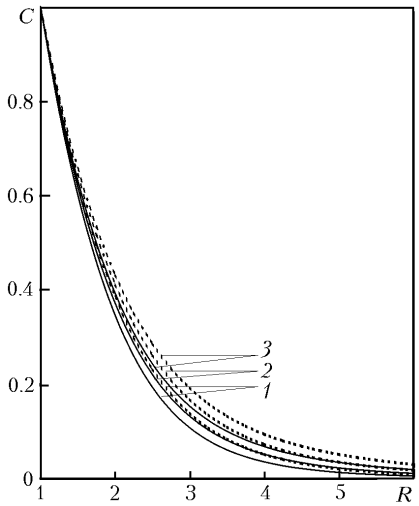

Fig. 2. Transient concentration profiles at $X = 1$ for Gr = Gc = 5, Sc = 0.6, and Pr = 0.71 (solid curves) and 7 (dashed curves) with different values of S: 1) $S = 0$; 2) 0.3; 3) 0.5.

$$\bar{\tau} = -\int_0^1 (\partial U/\partial R)_{R=1}\, dX\,, \tag{13}$$

$$\mathrm{Nu}_X = -X\,(\partial T/\partial R)_{R=1}\,, \tag{14}$$

$$\overline{\mathrm{Nu}} = -\int_0^1 (\partial T/\partial R)_{R=1}\, dX\,, \tag{15}$$

$$\mathrm{Sh}_X = -X\,(\partial C/\partial R)_{R=1}\,, \tag{16}$$

$$\overline{\mathrm{Sh}} = -\int_0^1 (\partial C/\partial R)_{R=1}\, dX\,. \tag{17}$$

The overbar denotes averages. The derivatives involved in Eqs. (12)–(17) are evaluated by the five-point approximation formula and integrals are estimated using the Newton–Cotes formula.

The local skin-friction profiles for different values of the Prandtl number and the stratification parameter are plotted in Fig. 3a as functions of the axial coordinate X. The local skin friction decreases as X increases. It is also seen that the local skin friction decreases with increasing stratification parameter and decreasing Prandtl number. The local Nusselt number for different Pr and S are presented in Fig. 3b. It is observed that the rate of heat transfer increases with the Prandtl number and the stratification parameter. The local Sherwood number for different values of S and Pr is shown in Fig. 3c for the steady-state case. It is seen that the local Sherwood number decreases with increasing values of the stratification parameter and the Prandtl number.

Average skin friction and Nusselt and Sherwood numbers at $X = 1$ are shown in Fig. 4 as functions of time for various parameters. Figure 4a shows that the skin friction increases with S. Initially, higher values of the average Nusselt and Sherwood numbers are observed and then they decrease with time (Figs. 4b and 4c). It is seen from Fig. 4b that the average heat-transfer rate increases with S, being the same for different S during the initial time period. As

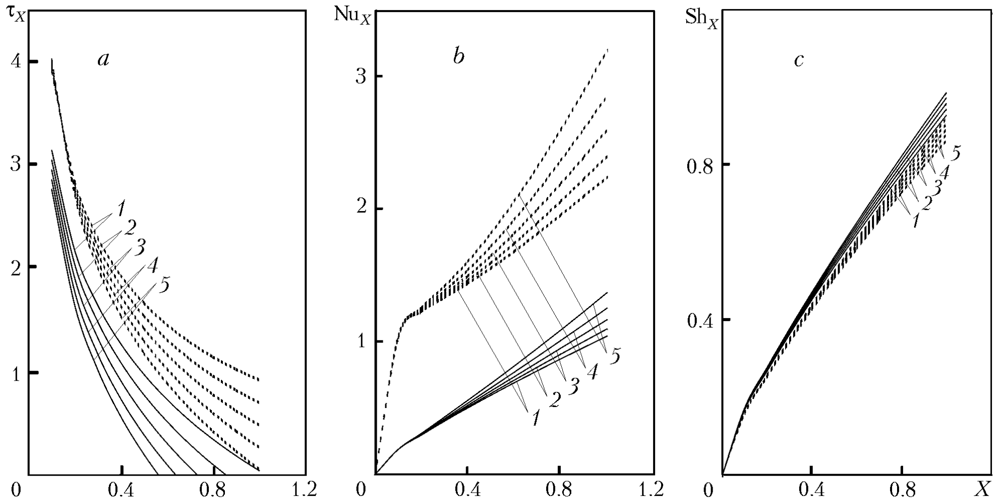

Fig. 3. Profiles of local values of skin friction (a), Nusselt number (b), and Sherwood number (c) at Gr = Gc = 5, Sc = 0.6, and Pr = 0.71 (solid curves) and 7 (dashed curves) with different values of S: 1) S = 0; 2) 0.1; 3) 0.2; 4) 0.3; 5) 0.4.

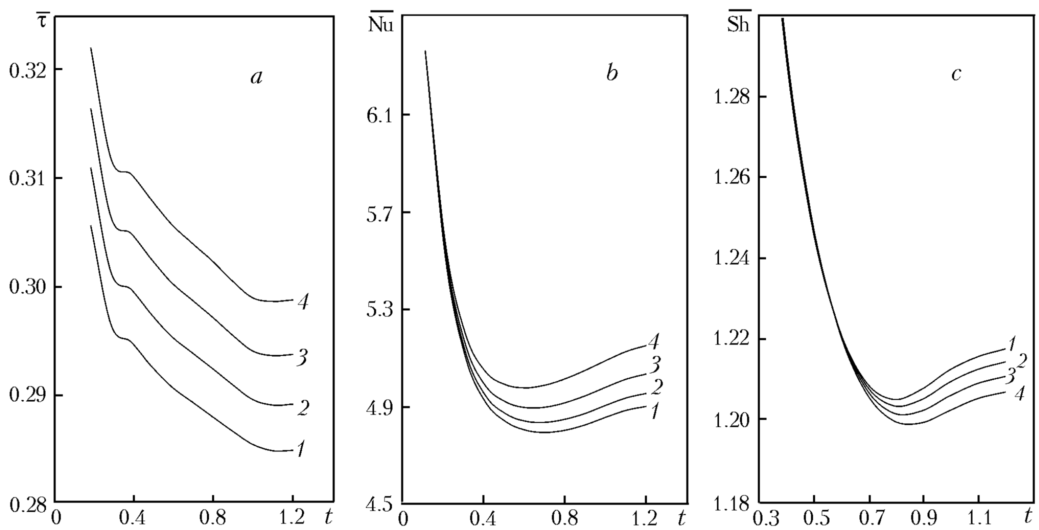

Fig. 4. Time dependence of average values of skin friction (a), Nusselt number (b), and Sherwood number (c) at X = 1 for Gr = Gc = 5, Sc = 0.6, and Pr = 7 with different values of S: 1) S = 0; 2) 0.1; 3) 0.2; 4) 0.3.

observed from Fig. 4c, the average Sherwood number initially is constant, too. This shows that there is only molecular diffusion during the initial period. Then the average Sherwood number decreases with increasing S.

NOTATION

C', species concentration of the fluid in the boundary layer; C'_∞, species concentration of the fluid away from a cylinder; C, relative species concentration of the fluid; D, binary diffusion coefficient; Gr, thermal Grashof number; Gc, mass Grashof number; g, acceleration due to gravity; $\overline{\mathrm{Nu}}$, average Nusselt number; Nu_X, local Nusselt number; Pr, Prandtl number; R, dimensionless radial coordinate; r, radial coordinate; r_0, radius of a cylinder; S, stratification parameter; Sc, Schmidt number; $\overline{\mathrm{Sh}}$, average Sherwood number; Sh_X, local Sherwood number; T, temperature of the fluid; T', temperature of the fluid in the boundary layer; $T'_{\infty,X}$, temperature of the fluid away from a cylinder; t, dimensionless time; t', time; U, V, dimensionless velocity components in X, R directions respectively; u, v, velocity components in x, r directions respectively; u_0, velocity of movement of a cylinder; X, dimensionless axial coordinate; x, axial coordinate; α, thermal diffusivity; β, volumetric coefficient of thermal expansion; β^*, volumetric coefficient of expansion with concentration; υ, kinematic viscosity; τ_X, local skin friction; $\bar{\tau}$, average skin friction. Subscripts: w, wall; ∞, free stream.

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